MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023

EME3046 –MECHANICS OF MATERIALS

(ME)

11 JULY 2023
9.00 a.m. - 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 13 pages including the cover page and appendix with 4 Questions.

2. Answer all questions.

3. All questions carry equal marks and the distribution of the marks for each question is given.

4. Write all your answers in the Answer Booklet provided.

5. Appendices are provided in pages 6-13.

Question 1:

Figure Q1 shows part of a complex machine (E = 300 GPa, ν = 0.30 and σ_U = 360MPa) that is subjected to various loads during operation. A 45 degrees strain rosette on the critical point A on the free surface of the machine part provides the following normal strain data: $\varepsilon_a = +650\mu m/m$, $\varepsilon_b = +475\mu m/m$ and $\varepsilon_c = -250\mu m/m$.

a) Determine the strain components ε_{xx}, ε_{yy}, and γ_{xy} at the critical point, by taking the gauge a to be oriented along the x-axis and gauge c to be oriented along the y-axis.

[3 marks]

b) Determine the stress components σ_{xx}, σ_{yy} and σ_{xy} at the critical point. Indicate the direction for the stress components obtained (tension/compression). Sketch a rectangular element and show these stresses.

[7 marks]

c) Determine the three principal strains (ε_1, ε_2, and ε_3), the direction of the principal planes and absolute maximum shearing strain.

[5 marks]

d) Determine the three principal stresses (σ_1, σ_2 and σ_3). Indicate direction for the principal stresses obtained (tension/compression).

[4 marks]

e) Determine the factor of safety in the design by applying the maximum principal:

i) Stress criterion **[3 marks]**

ii) Strain criterion **[3 marks]**

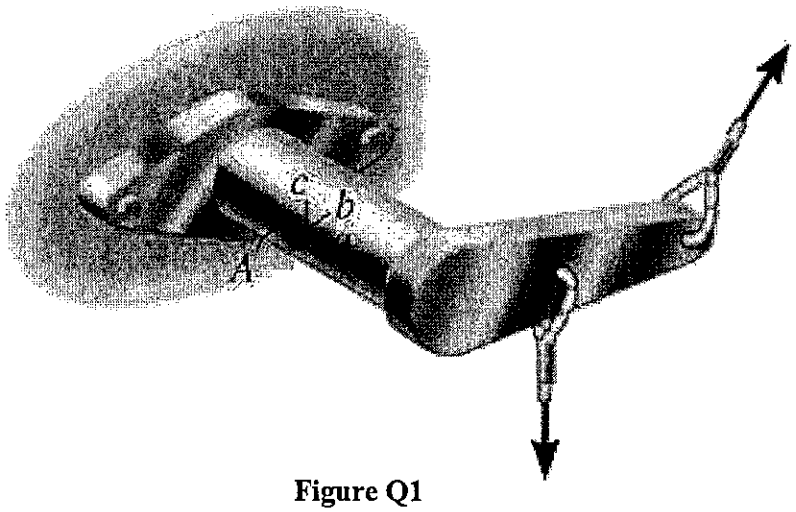


Figure Q1

Continued...

Question 2:

Figure Q2 shows a beam simply supported on the left and clamped on the right. A concentrated force P is applied at the position as shown. Given that $b > a$.

In terms of a, L, P, and flexural rigidity EI,

a) Draw the appropriate free body diagram(s) and write the equations of equilibrium. **[5 marks]**

b) Identify the unknowns and explain if this is a statically determinate problem. **[4 marks]**

c) Using the Castigliano's theorem, determine the reaction forces and moment at both ends of the beam. **[13 marks]**

d) Explain if the reaction force at the simply supported end will increase, decrease or remain if $a = b$. **[3 marks]**

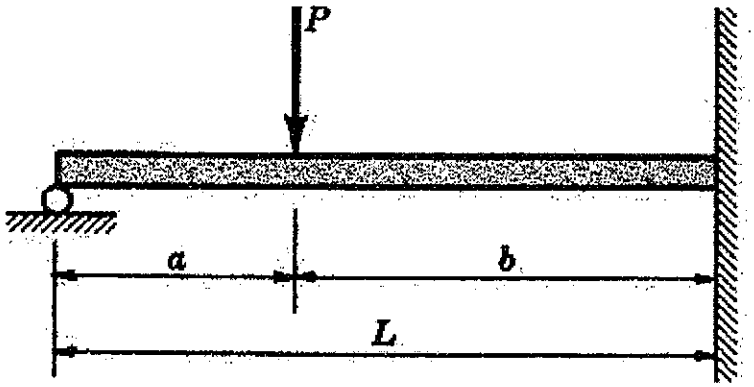


Figure Q2

Continued...

Question 3:

A multiply connected symmetrical steel tube (Figure Q1) resists a torque of 12 kNm. The wall thicknesses are $t_1 = t_2 = t_3 = 6$ mm and $t_4 = t_5 = 3$ mm. Given: $G = 80$ GPa, $A_1 = 0.079522$ m^2, $A_2 = 0.075$ m^2, $A_3 = 0.06$ m^2, $s_1 = 0.7069$ m, $s_2 = 0.2136$ m, $s_3 = 0.4272$ m, $s_4 = 0.45$ m, and $s_5 = 0.3$ m. A_i represents area bounded by the respective segment and s_i represents the circumference of segment with thickness t_i. Determine the shear flows (q_1, q_2 and q_3) and angle of twist.

[25 marks]

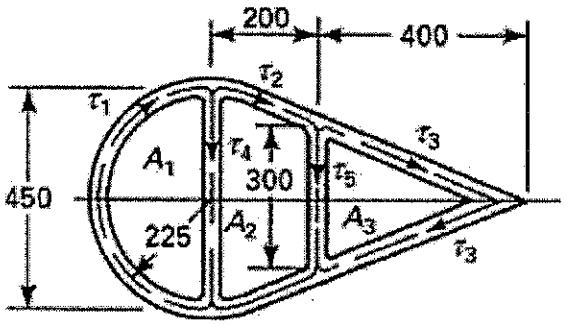


Figure Q3

Continued...

Question 4:

The two structural steel (E = 200 GPa, σ_Y = 250 MPa) members in Figure Q2a are 6 m long and have the cross section shown in Figure Q2b. These members can either experience in-plane buckling or out-of-plane buckling due to the load Q.

In case of in-plane buckling, the members are fixed-pinned and buckling takes place about the axis of minimum moment of inertia.

In case of out-of-plane buckling, the members are fixed-free and buckling takes place about the axis of maximum moment of inertia.

a) Draw free body diagram for one of the columns and show that the axial compressive load for each member, P is given as:

[3 marks]

$$P = \frac{Q}{\sqrt{2}}$$

b) Determine the magnitude of load Q that will cause the members to buckle (for both in-plane and out-of-plane) and identify which buckling will take place first. Use Euler formulas.

[14 marks]

c) AISI formulas for buckling are extension of the Euler formulas (by incorporating engineering considerations). Now, identify which AISI formula should be used for each case (in-plane buckling and out-of-plane buckling), and then determine the corresponding allowable stress, σ_{all} (for each case).

[8 marks]

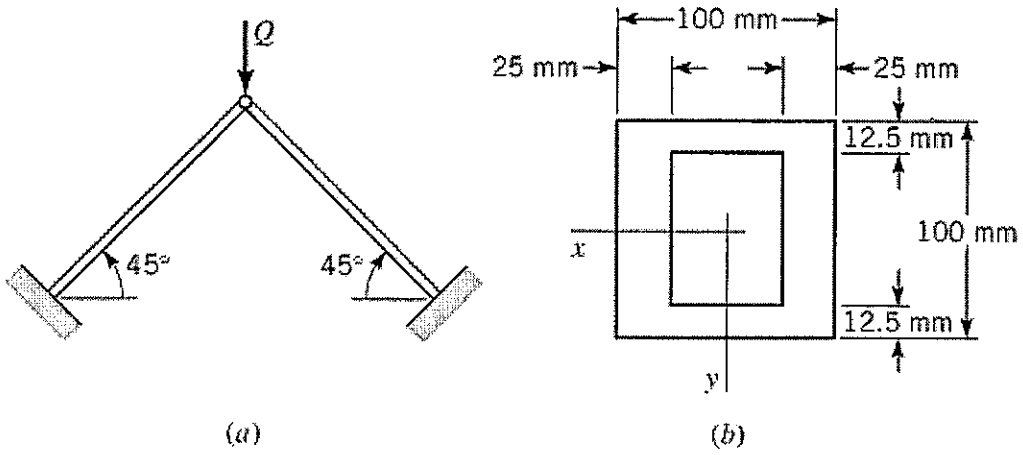


Figure Q4

Continued...

TRANSFORMATION OF STRESS

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$$

$$\sigma_{max,\,min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}}$$

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\,2}}$$

$$\sigma_{Ave} = \frac{\sigma_x + \sigma_y}{2}$$

TRANSFORMATION OF STRAIN

$$\varepsilon(\theta) = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_{y'} = \frac{\varepsilon_x + \varepsilon_y}{2} - \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta - \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\frac{\gamma_{x'y'}}{2} = -\frac{(\varepsilon_x - \varepsilon_y)}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

$$\varepsilon_{max,\,min} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\varepsilon_3 = -\frac{\nu}{1-\nu}(\varepsilon_1 + \varepsilon_2)$$

$$\tan 2\theta_s = -\left(\frac{\varepsilon_x - \varepsilon_y}{\gamma_{xy}}\right)$$

$$\frac{\gamma_{max}}{2}_{in-plane} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\varepsilon_{avg} = \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right)$$

GENERALIZED HOOKE'S LAW

$$\varepsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z)\right]$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_y + \nu(\varepsilon_x + \varepsilon_z)\right]$$

$$\sigma_z = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_z + \nu(\varepsilon_x + \varepsilon_y)\right]$$

Continued...

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

$$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$$

$$I_2 = \sigma_{xx}\sigma_{yy} + \sigma_{xx}\sigma_{zz} + \sigma_{yy}\sigma_{zz} - \sigma_{xy}^2 - \sigma_{xz}^2 - \sigma_{yz}^2$$

$$I_3 = \sigma_{xx}\left(\sigma_{yy}\sigma_{zz} - \sigma_{yz}\sigma_{yz}\right) - \sigma_{xy}\left(\sigma_{xy}\sigma_{zz} - \sigma_{xz}\sigma_{yz}\right) + \sigma_{xz}\left(\sigma_{xy}\sigma_{yz} - \sigma_{xz}\sigma_{yy}\right)$$

Plane stress

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\sigma_y\right] \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\sigma_x\right] \qquad \gamma_{yz} = 0$$

$$\varepsilon_z = -\frac{\nu}{E}\left[\left[\sigma_x + \sigma_y\right]\right] \qquad \gamma_{xz} = 0$$

Plane strain

$$\varepsilon_x = \frac{\left(1-\nu^2\right)}{E}\left(\sigma_x - \frac{\nu}{1-\nu}\sigma_y\right) \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\varepsilon_y = \frac{\left(1-\nu^2\right)}{E}\left(\sigma_y - \frac{\nu}{1-\nu}\sigma_x\right) \qquad \gamma_{yz} = 0$$

$$\varepsilon_z = 0 \qquad \gamma_{xz} = 0$$

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_y\right) \qquad \tau_{xy} = G\gamma_{xy}$$

$$\sigma_y = \frac{E}{1-\nu^2}\left(\varepsilon_y + \nu\varepsilon_x\right) \qquad \tau_{yz} = 0$$

$$\sigma_z = 0 \ (\textit{plane stress}) \qquad \tau_{xz} = 0$$

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\varepsilon_x + \nu\varepsilon_y\right] \qquad \tau_{xy} = G\gamma_{xy}$$

$$\sigma_y = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\varepsilon_x + (1-\nu)\varepsilon_y\right] \qquad \tau_{yz} = 0$$

$$\sigma_z = \frac{\nu E}{(1+\nu)(1-2\nu)}(\varepsilon_x + \varepsilon_y) \qquad \tau_{xz} = 0$$

Strain rosette

$$\varepsilon_x = \varepsilon_a \qquad \varepsilon_x = \varepsilon_a$$

$$\varepsilon_y = \varepsilon_c \qquad \varepsilon_y = \frac{1}{3}\left(2\varepsilon_b + 2\varepsilon_c - \varepsilon_a\right)$$

$$\gamma_{xy} = 2\varepsilon_b - \left(\varepsilon_a + \varepsilon_c\right) \qquad \gamma_{xy} = \frac{2}{\sqrt{3}}\left(\varepsilon_b - \varepsilon_c\right)$$

Continued...

Failure criteria

$$(SF)\sigma_{max} = (SF)\sigma_1 = \sigma_u$$

$$(SF) = \frac{\sigma_u}{\sigma_1 - \nu\sigma_2} \qquad SF = \frac{\sqrt{2}Y}{[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}}$$

$$(SF)\tau_{max} = \frac{Y}{2}$$

COLUMNS BUCKLING

$$y_{max} = e\left[\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right) - 1\right]$$

$$M = P\,(y_m + e)$$

radius of gyration

$$r_i = \sqrt{\frac{I_i}{A}} \;:\; i = x, y, z$$

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L_e/r)^2}$$

$$\left(\frac{Le}{r}\right)_c = \pi\sqrt{\frac{E}{\sigma_P}}$$

$$\boxed{\sigma_{max} = \frac{P}{A}\left(1 + \frac{ec}{r^2}\right)}$$

$$\sigma_{cr} = \frac{P_t}{A} = \frac{\pi^2 E_t}{(L_e/r)^2}$$

$$\sigma_{cr} = \sigma_y - \frac{1}{E}\left(\frac{\sigma_y}{2\pi}\frac{L_e}{r}\right)^2$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right]$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L_e}{2r}\sqrt{\frac{P}{AE}}\right)\right]$$

$$\boxed{P_{cr} = \sigma_{cr}A = \sigma_y A\left[1 - \frac{\sigma_y(L_e/r)^2}{4\pi^2 E}\right]}$$

Continued...

Structural Steel Columns

$$\sigma_{all} = \frac{\sigma_y}{n_s}\left[1 - \frac{1}{2}\left(\frac{L_e/r}{C_c}\right)^2\right] \qquad \left(\frac{L_e}{r} < C_c\right)$$

$$\sigma_{all} = \frac{\pi^2 E}{1.92(L_e/r)^2} \qquad \left(C_c \le \frac{L_e}{r} \le 200\right)$$

$$C_c = \sqrt{2\pi^2 E/\sigma_y}$$

$$n_s = \frac{5}{3} + \frac{3}{8}\left(\frac{L_e/r}{C_c}\right) - \frac{1}{8}\left(\frac{L_e/r}{C_c}\right)^3$$

Timber Columns of Rectangular Cross Section

$$\sigma_{all} = 1.2\,\text{ksi} = 8.27\,\text{MPa} \qquad \left(\frac{L_e}{d} \le 11\right)$$

$$\sigma_{all} = 1.2\left[1 - \frac{1}{3}\left(\frac{L_e/d}{26}\right)^2\right]\text{ksi} \qquad \left(11 < \frac{L_e}{d} \le 26\right)$$

$$\sigma_{all} = \frac{0.3E}{(L_e/d)^2} \qquad \left(26 < \frac{L_e}{d} \le 50\right)$$

Aluminum Alloy 6061-T6 Columns

Equation 11.27: $\sigma_{all} = 19\,\text{ksi} = 130\,\text{MPa} \qquad \left(\frac{L_e}{r} \le 9.5\right)$

Equation 11.28: $\sigma_{all} = \left[20.2 - 0.126\left(\frac{L_e}{r}\right)\right]\text{ksi}$

$$= \left[140 - 0.87\left(\frac{L_e}{r}\right)\right]\text{MPa} \qquad \left(9.5 < \frac{L_e}{r} < 66\right)$$

Equation 11.29: $\sigma_{all} = \frac{51{,}000\,\text{ksi}}{(L_e/r)^2}$

$$= \frac{350 \times 10^3}{(L_e/r)^2}\,\text{MPa} \qquad \left(\frac{L_e}{r} \ge 66\right)$$

$$q_i = \sum_{j=1}^{m}\left(\int \frac{f_j}{E_j A_j}\frac{\partial f_j}{\partial F_i}dz + \int \frac{M_j}{E_j I_j}\frac{\partial M_j}{\partial F_i}dz + \int \frac{k V_j}{G_j A_j}\frac{\partial V_j}{\partial F_i}dz + \int \frac{T_j}{G_j J_j}\frac{\partial T_j}{\partial F_i}dz\right)$$

$$\theta_i = \sum_{j=1}^{m}\left(\int \frac{f_j}{E_j A_j}\frac{\partial N_j}{\partial M_i}dz + \int \frac{M_j}{E_j I_j}\frac{\partial M_j}{\partial M_i}dz + \int \frac{k V_j}{G_j A_j}\frac{\partial V_j}{\partial M_i}dz + \int \frac{T_j}{G_j J_j}\frac{\partial T_j}{\partial M_i}dz\right)$$

Continued...

$\gamma_{max} = c\phi/L \quad \tau_{max} = Tc/J \quad \gamma_{max} = \tau_{max}/G \quad \phi = TL/GJ$

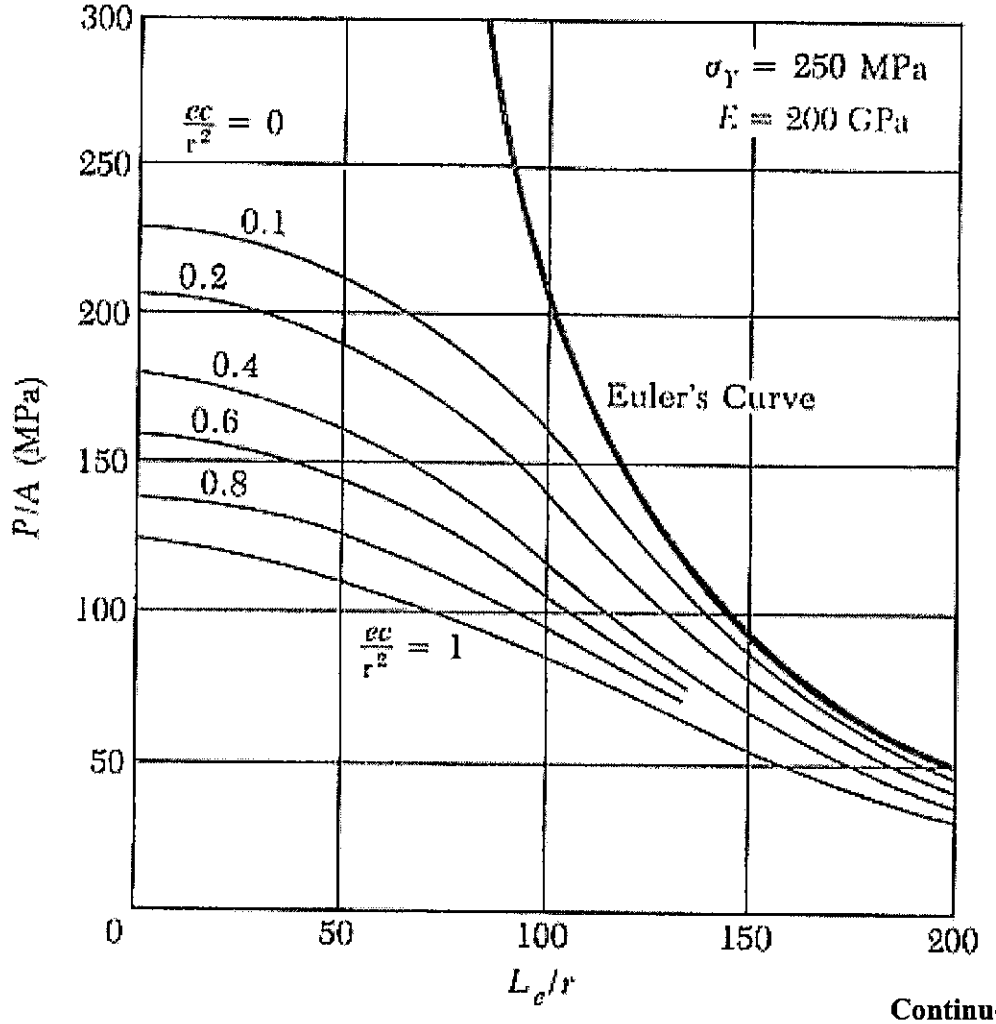
$$T_{EP} = \frac{4}{3}T_Y\left(1 - \frac{1}{4k^3}\right) \qquad T_{EP} = \frac{\pi\tau_Y}{12}(D_o^3 - D_i^3)$$

$$T_{EP} = \frac{4}{3}T_Y = \frac{\pi}{12}\tau_Y D^3 \qquad \frac{\gamma_Y}{\gamma_{max}} = \frac{r_Y}{b} = \frac{1}{k} \qquad q = \tau t$$

$$\theta = \frac{1}{2GA}\oint \tau dl = \frac{q}{2GA}\oint \frac{dl}{t} \qquad T = 2\sum_{i=1}^{N} A_i \frac{z_i}{c} = 2\sum_{i=1}^{N} A_i q_i$$

$$\theta = \frac{1}{2GA_i}\oint \frac{q_i - \acute{q}}{t} dl, i = 1, 2, \ldots, N$$

FIG A. Load per unit area. P/A, causing yield in column according to secant formula.



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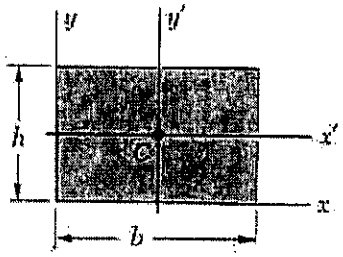
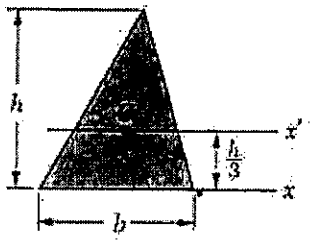
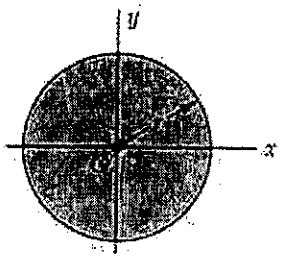
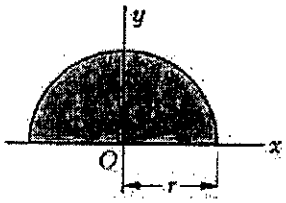
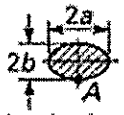
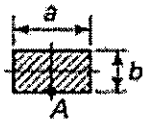
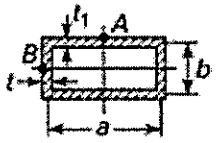
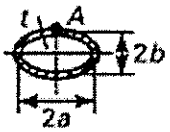
Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{36}bh^3$ $I_x = \frac{1}{12}bh^3$
Circle		$I_x = I_y = \frac{1}{4}\pi r^4$ $J_O = \frac{1}{2}\pi r^4$
Semicircle		$I_x = I_y = \frac{1}{8}\pi r^4$ $J_O = \frac{1}{4}\pi r^4$

FIG B. Moments of inertia of common geometric shapes.

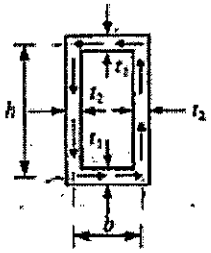
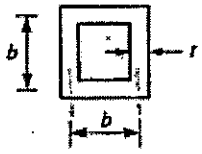
Continued...

TABLE 6.2 *Shear Stress and Angle of Twist of Various Members in Torsion*

Cross section	*Maximum shearing stress*	*Angle of twist per unit length*
 For circular bar: $a = b$	$\tau_A = \dfrac{2T}{\pi ab^2}$	$\theta = \dfrac{(a^2 + b^2)T}{\pi a^3 b^3 G}$
Equilateral triangle	$\tau_A = \dfrac{20T}{a^3}$	$\theta = \dfrac{46.2T}{a^4 G}$
	$\tau_A = \dfrac{T}{\alpha ab^2}$	$\theta = \dfrac{T}{\beta ab^3 G}$
	a/b	β / α
	1.0	0.141 / 0.208
	1.5	0.196 / 0.231
	2.0	0.229 / 0.246
	2.5	0.249 / 0.256
	3.0	0.263 / 0.267
	4.0	0.281 / 0.282
	5.0	0.291 / 0.292
	10.0	0.312 / 0.312
	∞	0.333 / 0.333
	$\tau_A = \dfrac{T}{2abt_1}$ $\tau_B = \dfrac{T}{2abt}$	$\theta = \dfrac{(at + bt_1)T}{2tt_1 a^2 b^2 G}$
 For circular tube: $a = b$	$\tau_A = \dfrac{T}{2\pi abt}$	$\theta = \dfrac{\sqrt{2(a^2 + b^2)}T}{4\pi a^2 b^2 tG}$
Hexagon	$\tau_A = \dfrac{5.7T}{a^3}$	$\theta = \dfrac{8.8T}{a^4 G}$

Continued...

Table 9-1 St. Venant Torsion

	Shear Stress f_{sT} (ksi)	Torsional Constant J (in^4)
Closed sections		
Round bar	$\frac{Tr}{J}$	$\frac{\pi R^4}{2}$
Round tube	$\frac{Tr}{J}$	$\frac{\pi}{2}(R_o^4 - R_i^4)$
 Rectangular tube	$\frac{T}{2bht_1}$ $\frac{T}{2bht_2}$	$\frac{2t_1t_2b^2h^2}{bt_2 + ht_1}$
 Square tube	$\frac{T}{2b^2t}$	tb^3
Rectangular bar	$\frac{Tt}{J}$	$\frac{bt^3}{3}$ (approx.)
Open sections	$\frac{Tt_i}{J}$	$\frac{\Sigma bt^3}{3}$ (approx.; for exact values, see AISC LRFD Manual, pp. 1-133—1-161)

End of Page.